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Analysis of threshold field effects in smectic C phases

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A qualitative analysis of the threshold behaviour of plane smectic C layers in external fields is made. All possible relative orientations between the liquid crystal, the boundaries and the field vector are considered. Arbitrary material constants are assumed in the uniaxial approximation. The conditions for first- and second-order transitions are given.

1. Introduction

Field effects in smectic C phases, considered first by Rapini [1], are in practice limited to field-induced reorientation of the director. The distortions of the smectic layers are too small to be detected. Deformation of the director field is possible owing to the rotation of the director around the normal to the smectic layers. This effect has been considered both theoretically [2–5] and experimentally [3, 5]. In the theoretical description an approximation concerning the elastic constants was used: $B_1 = B_2 = B_3$, $B_{13} = 0$. In this paper the more general case is considered; arbitrary values of the elastic constants are assumed and all possible positions of the smectic layers are considered. Application of a magnetic field is treated. The most practical case of the field vector normal to the boundary plates is discussed separately and illustrated by means of the electric field. It is assumed that the material is uniaxial and the corresponding anisotropies $\chi_a = \chi_\parallel - \chi_\perp$ and $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$ are small. Both signs of the anisotropies are considered.

The threshold character of the field-induced reorientation has been investigated by means of an analysis of the expansion of the layer free energy in a power series in the director deformation angle. The proper truncation of the series was guaranteed by application of theorems from catastrophe theory [6]. According to Thom's theorem, any family of smooth functions of n variables and r parameters is equivalent to one of a few archetypal forms. All but two of these involve a dependence on parameters and are called 'catastrophes'. The catastrophe predicts the number and kind of critical points of the function considered, i.e. the points at which the first derivative vanishes. The behaviour of the system depends on the degeneracy of the critical point, i.e. on the number of the successive higher derivatives that are zero at this point. The most interesting phenomena occur in the vicinity of the degenerate critical point. Thresholds or discontinuities are the characteristic features of this behaviour. If the potential energy of the system has the form of a catastrophe, we can find the character of the equilibrium states and their evolution under variation of the parameters. There are seven elementary catastrophes resulting if $r \leqslant 4$. In this paper the so-called 'butterfly' catastrophe is used, its properties have been described in a previous paper [7], where the same approach was applied to some cases of field effects in nematic layers. The results are qualitative, owing to the topological character of catastrophe theory. They are valid only locally, i.e. the number of solutions and their disposition in the vicinity of the critical point can be revealed. The numerical values are only approximate.

In §2 the geometry of the system is defined and field-induced deformations are considered. The results are described in §3. Section 4 contains some concluding remarks.

2. The geometry of the system

The geometry of the system is shown in figure 1. The flat smectic layers with uniform director orientation and tilt angle ω are contained between two parallel plates separated by a distance d. Their positions are defined by the normal $\mathbf{K}$, which always lies in the (y, z) plane and makes an angle β with the substrate. The director is confined to the surface of a cone and is determined by the angle $\phi(z)$, its initial value is ϕ_0. The difference $\phi(z) - \phi_0 = \xi(z)$ is the measure of the director deviation. It is useful to represent β as a sum of two angles: θ, between the projection of $\mathbf{n}_0$ on the (y, z) plane and the substrate; and α, between this projection and $\mathbf{K}$. The three angles ϕ_0, α and ω are related by

$$\cos \phi_0 = \frac{\tan \alpha}{\tan \omega}. \tag{1}$$

A magnetic field of strength H is applied in the direction determined by the angles γ and δ, which are measured relative to the (y, z) plane and $\mathbf{K}$ respectively.

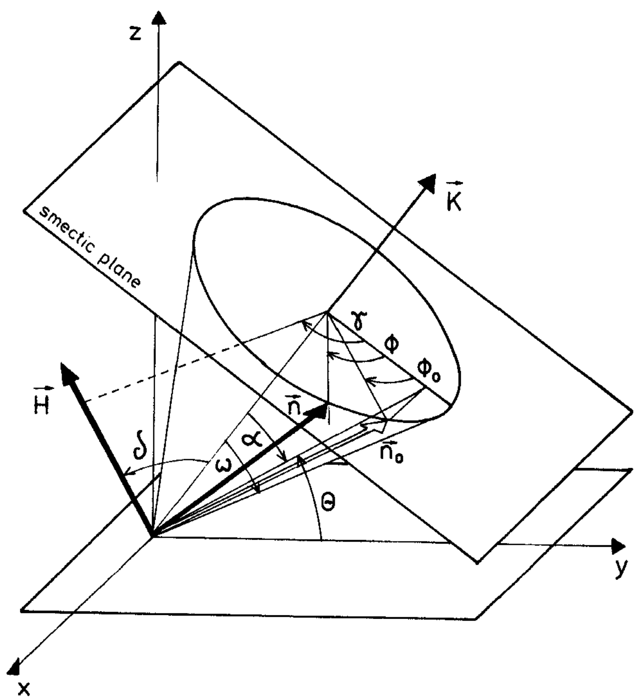

Figure 1. The geometry of the smectic C layer in an external field.

In this geometry the free-energy density is given by

$$g = \frac{B_3}{2}\left\{(b_1 \cos^2\phi \cos^2\beta + b_2 \sin^2\phi \cos^2\beta + \sin^2\beta - b_{13}\cos\phi \sin 2\beta)\left(\frac{\partial\phi}{\partial z}\right)^2 - \frac{\pi^2 h}{d\sin^2\omega}[\sin\omega \sin\delta \cos(\phi-\gamma) + \cos\omega\cos\delta]^2\right\}, \tag{2}$$

where reduced quantities are used: $b_1 = B_1/B_3, b_2 = B_2/B_3, b_{13} = B_{13}/B_3, h = (H/H_0)^2$ and $H_0 = (\pi/d \sin\omega)(B_3/\chi_a)^{1/2}$. Small deformations are assumed and so the director distribution in the slab can be approximated by the function

$$\phi(z) = \phi_0 + \xi_m \cos(\pi z/d), \tag{3}$$

where ξ_m is the amplitude of the deformation in the middle plane of the slab. Using this result, we can expand the free energy density in equation (2) in a Taylor series in the vicinity of $\xi_m = 0$. Integration over z gives the free energy per the unit area of the sample (accurate to within a constant):

$$G = \sum_{i=1}^{\infty} a_i \xi_m^i, \tag{4}$$

where the coefficients a_i are

$$a_1 = \frac{\pi B_3 h}{d}\frac{\sin\delta}{\sin\omega}[\sin\omega\sin\delta\sin 2(\phi_0-\gamma) + 2\cos\omega\cos\delta\sin(\phi_0-\gamma)], \tag{5}$$

$$a_2 = \frac{\pi^2 B_3}{4d}\left\{b_1\cos^2\phi_0\cos^2\beta + b_2\sin^2\phi_0\cos^2\beta + \sin^2\beta - b_{13}\cos\phi_0\sin 2\beta + \frac{h}{2}\frac{\sin\delta}{\sin\omega}[2\sin\omega\sin\delta\cos 2(\phi_0-\gamma) + 2\cos\omega\cos\delta\cos(\phi_0-\gamma)]\right\}, \tag{6}$$

and for $i \geqslant 3$

$$a_i = \frac{\pi^2 B_3}{2i[(i-2)!!]^2 d}\left\{(-1)^{i/2}[2^{i-3}(b_2-b_1)\cos 2\phi_0\cos^2\beta + b_{13}\cos\phi_0\sin 2\beta] + \frac{(-1)^{(i+2)/2}h}{i}\frac{\sin\delta}{\sin\omega}[2^{i-1}\sin\omega\sin\delta\cos 2(\phi_0-\gamma) + 2\cos\omega\cos\delta\cos(\phi_0-\gamma)]\right\} \tag{7}$$

if i is even, and

$$a_i = \frac{\pi B_3}{i[(i-2)!!]^2 d}\left\{(-1)^{(i+1)/2}[2^{i-3}(b_2-b_1)\sin 2\phi_0\cos^2\beta + b_{13}\sin\phi_0\sin 2\beta] + \frac{(-1)^{(i-1)/2}h}{i}\frac{\sin\delta}{\sin\omega}[2^{i-1}\sin\omega\sin\delta\sin 2(\phi_0-\gamma) + 2\cos\omega\cos\delta\sin(\phi_0-\gamma)]\right\} \tag{8}$$

if i is odd. These coefficients depend on the material parameters b_1, b_2, b_{13}, ω and the angles ϕ_0, β, as well as the field strength and direction h, δ, γ. It is possible to find some relations between these parameters, and for any physically acceptable values of them the coefficients $a_1, \ldots, a_5$ vanish, although a_6 is non-zero. This means that there exist degenerate critical points at $\xi_m = 0$ for each particular set of parameters, since a_i is proportional to the ith derivative of the free-energy function.

According to the theorems of catastrophe theory, terms of order higher than six in expansion (4) can be disregarded. The energy G, truncated in this way, can be written in the form of the butterfly catastrophe

$$f = \tfrac{1}{6}x^6 + \tfrac{1}{4}ax^4 + \tfrac{1}{3}bx^3 + \tfrac{1}{2}cx^2 + dx, \tag{9}$$

where the transformation of variables

$$x = (6\,|\,a_6\,|)^{1/6}\left(\xi_m + \frac{a_5}{6a_6}\right) \tag{10}$$

is used and the coefficients are

$$a = \frac{12a_4a_6 - 5a_5^2}{3a_6(6\,|\,a_6\,|)^{2/3}}, \tag{11 a}$$

$$b = \frac{5a_5^3 - 18a_4a_5a_6 + 27a_3a_6^2}{9a_6^2(6\,|\,a_6\,|)^{1/2}}, \tag{11 b}$$

$$c = \frac{-5a_5^4 + 24a_4a_5^2a_6 - 72a_3a_5a_6^2 + 144a_2a_6^3}{72a_6^3(6\,|\,a_6\,|)^{1/3}} \tag{11 c}$$

$$d = \frac{a_5^5 - 6a_4a_5^3a_6 + 27a_3a_5^2a_6^2 - 108a_2a_5a_6^3 + 324a_1a_6^4}{324a_6^4(6\,|\,a_6\,|)^{1/6}}. \tag{11 d}$$

The application of this standardization is useful only if $b = d = 0$, i.e. if $a_1 = a_3 = a_5 = 0$. The catastrophe manifold then reduces to the form

$$x = 0, \tag{12}$$

$$x^4 + ax^2 + c = 0, \tag{13}$$

where

$$a = 4a_4/(6\,|\,a_6\,|)^{2/3}, \tag{14}$$

$$c = 2a_2/(6\,|\,a_6\,|)^{1/3}, \tag{15}$$

$$x = (6\,|\,a_6\,|)^{1/6}\xi_m. \tag{16}$$

The surfaces defined by equations (12) and (13) represent the ξ_m values for which the total free energy G has extremes. For every set of parameters of the system, a point in the (a, c) plane is defined. Under variation of the field strength these points form a trajectory, which determines the required solutions. The threshold field strength is reached if c becomes zero. If the trajectory intersects the line $c = 0$ for $a \geqslant 0$ then the transition is second-order. In the opposite case a first-order transition occurs. In general, if b and d are non-zero, it is more convenient to work with the untransformed expansion.

It may happen that $a_6 < 0$. The resulting catastrophe is called the 'dual butterfly'. Its properties are analogous to the standard one, but minima and maxima are interchanged and the senses of the a, b, c and d axes are reversed.

3. Results

The deformation of the director field has threshold character if the total free energy in the undistorted state is extreme, i.e. if $a_1 = 0$ for any field strength. This defines two situations in which the critical point is realized:

$$\phi_0 = \gamma \tag{17}$$

and

$$\cos(\phi_0 - \gamma) = -\cot\omega \cot\delta. \tag{18}$$

The system remains undeformed until the threshold field, determined by the condition $a_2 = 0$, is reached. The shape of solutions ξ_m as a function of the external field in the neighbourhood of the threshold depends on the values of further coefficients in the expansion and can be of five types.

1. Symmetrical without a bistability range if $a_3 = a_5 = 0$ for any field together with $a_4 > 0$ at the threshold, which is equivalent to

$$b = d = 0, \quad a \geqslant 0 \quad \text{for } c = 0. \tag{19}$$

The transition is second-order. Both signs of the deformation are equivalent. Curve 1 in figure 2 illustrates such a result.

2. Symmetrical with a bistability range if

$$b = d = 0, \quad a < 0 \quad \text{for } c = 0. \tag{20}$$

The transition is first-order and hysteresis is possible (see curve 2 in figure 2).

3. Asymmetrical with a bistability range and one deformed state possible at the threshold if $a_3 \neq 0$ (see figure 3, curve 1).
4. Asymmetrical with a bistability range and two equally probable deformed states if $a_3 = 0$, $a_4 < 0$ and $a_5 \neq 0$ at the threshold (see figure 4 (*a*)).
5. Asymmetrical without bistability if $a_3 = 0$, $a_4 \geqslant 0$ and $a_5 \neq 0$ at the threshold. The transition is second-order in this particular case (see figure 4 (*b*)).

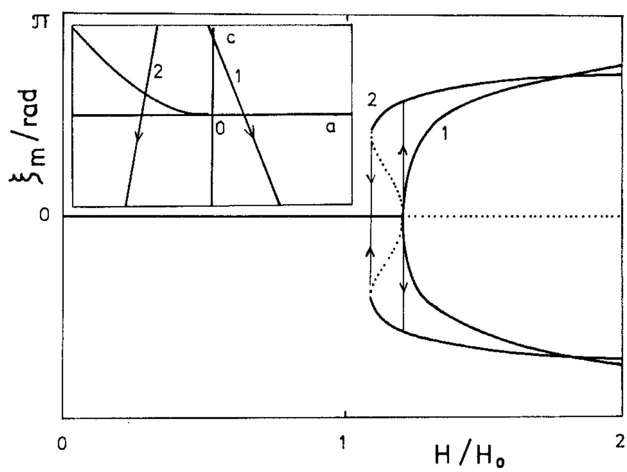

Figure 2. The deformation angle ξ_m as a function of the external field strength; $b_1 = 1{\cdot}4$, $b_2 = 1{\cdot}2$, $b_{13} = 0{\cdot}2$, $\omega = 15°$ (material constants are common to all figures). $\gamma = \phi_0 = 0°$ and $\theta = 0°$. Curve 1: $\delta = -15°$, type 1. Curve 2: $\delta = -60°$, type 2. The insert shows the corresponding trajectories in the (a, c) plane. The arrows indicate increasing field strength. Full lines represent minima, dotted lines maxima.

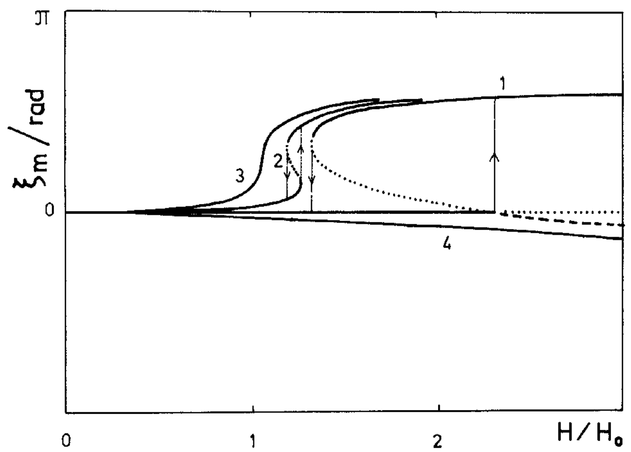

Figure 3. The deformation angle ξ_m as a function of the external field strength; $\gamma = 0°$ and $\phi_0 = 30°$. Curve 1: $\theta = 0°$ and $\delta = -76{\cdot}94°$ ($\mathbf{H} \perp \mathbf{n}_0$), type 3. Curve 2: $\theta = 2°$ and $\delta = -74{\cdot}94°$. Curve 3: $\theta = 5°$ and $\delta = -71{\cdot}94°$. Curve 4: $\theta = -2°$ and $\delta = -78{\cdot}94°$. The field vector is perpendicular to the boundary plates. The dashed line represents unavailable minima. Inessential maxima are not shown.

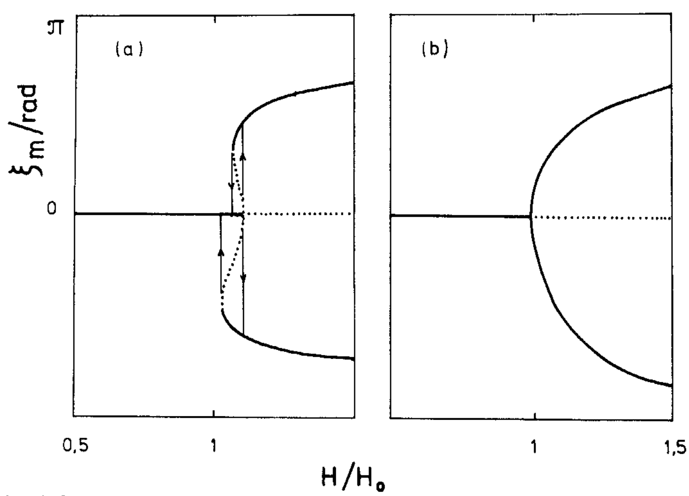

Figure 4. The deformation angle ξ_m as a function of the external field strength; $\gamma = \phi_0 = 45°$ and $\theta = 24{\cdot}53°$. (*a*) $\delta = -60°$, type 4. (*b*) $\delta = -20°$, type 5.

3.1. *Deformations in a magnetic field*

3.1.1. *Positive diamagnetic anisotropy*

(*a*) $\phi_0 = \gamma$. If $\phi_0 = \gamma$, the three vectors $\mathbf{H}$, $\mathbf{K}$ and $\mathbf{n}_0$ lie in the same plane. In general, a_3 and a_5 do not vanish and a deformation of type 3 is realized. The threshold field is given by

$$h_c = -\frac{\sin \omega\,(b_1 \cos^2 \phi_0 \cos^2 \beta + b_2 \sin^2 \phi_0 \cos^2 \beta + \sin^2 \beta - b_{13} \cos \phi_0 \sin 2\beta)}{\sin \delta \cos(\delta - \omega)}. \tag{21}$$

The condition $h_c > 0$ yields restrictions on δ: the transition takes place only if

$$\omega - \tfrac{1}{2}\pi < \delta < 0 \quad \text{or} \quad \omega + \tfrac{1}{2}\pi < \delta < \pi. \tag{22}$$

The additional condition

$$\tan \beta = \frac{b_1 - b_2}{b_{13}} \cos \phi_0 \tag{23}$$

together with

$$\min(\delta_c, \omega - \tfrac{1}{2}\pi) < \delta < \max(\delta_c, \omega - \tfrac{1}{2}\pi) \tag{24}$$

give a transition of type 4. If equation (23) is satisfied together with the relation opposite to (24),

$$\max(\delta_c, \omega - \tfrac{1}{2}\pi) \leqslant \delta < \tfrac{1}{2}\pi \quad \text{or} \quad -\tfrac{1}{2}\pi < \delta \leqslant \min(\delta_c, \omega - \tfrac{1}{2}\pi), \tag{25}$$

or equivalently

$$\max(\delta_c - \pi, \omega - \tfrac{3}{2}\pi) \leqslant \delta < -\tfrac{1}{2}\pi \quad \text{or} \quad \tfrac{1}{2}\pi < \delta \leqslant \min(\delta_c + \pi, \omega + \tfrac{1}{2}\pi), \tag{25$'$}$$

then type 5 is realized. In these expressions δ_c is determined by

$$\begin{aligned}\tan \delta_c = {} & -\{[4(b_2 - b_1)\cos 2\phi_0 + b_2 \sin^2 \phi_0 - b_1 \cos^2 \phi_0] \cos^2 \beta \\ & + \sin^2 \beta + b_{13} \cos \phi_0 \sin 2\beta\} \cot \omega \\ & \times [4 \cos^2 \beta\, (b_2 \cos^2 \phi_0 + b_1 \sin^2 \phi_0) + 4 \sin^2 \beta - 2b_{13} \cos \phi_0 \sin 2\beta]^{-1}.\end{aligned} \tag{26}$$

If equation (23) is valid, this formula takes the form

$$\tan \delta_c = -\frac{(4b_2 - 3b_1)\cos^2 \beta + \sin^2 \beta + b_{13} \sin 2\beta}{4b_2 \cos^2 \beta + 4 \sin^2 \beta - 2b_{13} \cos \phi_0 \sin 2\beta} \cot \omega \tag{27}$$

As a consequence of requirements (22), the inequalities (25) and (25′) are reduced to

$$\max(\delta_c, \omega - \tfrac{1}{2}\pi) \leqslant \delta < 0 \quad \text{or} \quad \max(\delta_c - \pi, \omega - \tfrac{3}{2}\pi) \leqslant \delta < -\pi. \tag{28}$$

Symmetrical solutions can be obtained in one of the following situations

(i) $\phi_0 = 0$;
(ii) $\beta = \frac{1}{2}\pi$;
(iii) $\phi_0 = \frac{1}{2}\pi$ and $\beta = 0$;
(iv) $\phi_0 = \frac{1}{2}\pi$ and $b_{13} = 0$;
(v) $b_1 = b_2$ and $\beta = 0$;
(vi) $b_1 = b_2$ and $b_{13} = 0$.

The transition is of type 1 if the relations (28) are satisfied. A discontinuous deformation of type 2 occurs if inequality (24) is true. In case (i) δ_c is determined by equation (27). For cases (ii)–(vi) this equation takes the simple form

$$\tan \delta_c = -\tfrac{1}{4} \cot \omega, \tag{29}$$

which was obtained in [3] in the single-constant approximation.

(*b*) $\mathbf{H} \perp \mathbf{n}_0$. If the vector $\mathbf{H}$ is perpendicular to the initial director $\mathbf{n}_0$ then relation (18) is obeyed. The threshold field is given by

$$h_c = \frac{b_1 \cos^2 \phi_0 \cos^2 \beta + b_2 \sin^2 \phi_0 \cos^2 \beta + \sin^2 \beta - b_{13} \cos \phi_0 \sin 2\beta}{\sin^2 \delta \sin^2 (\phi_0 - \gamma)}. \tag{30}$$

In general, a type 3 deformation occurs. However, if the following rather complicated relation is satisfied,

$$\cot(\phi_0 - \gamma) = \frac{(b_2 - b_1)\sin 2\phi_0 + 2\tan\beta\sin\phi_0}{2(b_1\cos^2\phi_0 + b_2\sin^2\phi_0 + \tan^2\beta - 2b_{13}\tan\beta\cos\phi_0)}, \quad (31)$$

then types 4 or 5 are realized. Type 4 can occur if

$$\tan^2\beta\,[\sin^2(\phi_0 - \gamma) - 3] + 2\tan\beta\, b_{13}\cos\phi_0\,[3 - 5\sin^2(\phi_0 - \gamma)]$$
$$+ [3(b_1 + b_2) + b_1\sin^2\phi_0 + b_2\cos^2\phi_0]\sin^2(\phi_0 - \gamma)$$
$$- 3(b_1\cos^2\phi_0 + b_2\sin^2\phi_0) < 0, \quad (32)$$

and type 5 in the opposite case.

Symmetrical solutions can be obtained in the following configurations if $\delta = \frac{1}{2}\pi$:

(vii) $\beta = \frac{1}{2}\pi$ and $\phi_0 - \gamma = \frac{1}{2}\pi$ ($h_c = 1$);
(viii) $\beta = 0$ and $\phi_0 = \frac{1}{2}\pi$ and $\gamma = 0$ ($h_c = b_2$);
(ix) $\phi_0 = \frac{1}{2}\pi$ and $\gamma = 0$ and $b_{13} = 0$ ($h_c = b_2\cos^2\beta + \sin^2\beta$);
(x) $\beta = 0$ and $\phi_0 - \gamma = \frac{1}{2}\pi$ and $b_1 = b_2 = b_0$ ($h_c = b_0$);
(xi) $\phi_0 - \gamma = \frac{1}{2}\pi$, $b_1 = b_2 = b_0$ and $b_{13} = 0$ ($h_c = b_0\cos^2\beta + \sin^2\beta$);
(xii) $\phi_0 = 0$ and $\gamma = \frac{1}{2}\pi$ ($h_c = b_1\cos^2\beta + \sin^2\beta - b_{13}\sin^2\beta$).

In cases (vii)–(xi) the transition is of type 1 since $a > 0$ for $c = 0$. In case (xii) type 2 is possible if

$$\tan^2\beta - b_{13}\tan\beta + b_2 < 0. \quad (33)$$

This relation results from conditions (20) and can take place if the rather unlikely inequality

$$b_{13}^2 > 4b_2 \quad (34)$$

is obeyed.

3.1.2. *Negative diamagnetic anisotropy*

Negative diamagnetic anisotropy can be introduced into the coefficients of the expansion by changing the sign before the reduced field h. As we have seen, the expansion can be limited to sixth degree.

The configurations with $\mathbf{H} \perp \mathbf{n}_0$ are stable and give negative values of h_c. Therefore the critical points at $\xi_m = 0$, which should be considered, occur if $\phi_0 = \gamma$. The threshold field can be obtained from equation (21) after a change of sign. The deformation occurs only if $0 < \delta < \omega + \frac{1}{2}\pi$ or $-\pi < \delta < \omega - \frac{1}{2}\pi$. The coefficient a_4 is always positive. In general, the deformation is of type 3; type 5 can be realized if equation (23) is obeyed.

Symmetrical solutions are present in configurations (i)–(vi). All of them are continuous.

3.2. *Application of an electric field*

In the case when an electric field is present the boundary plates play the role of electrodes, therefore $\gamma = 0$ and $\delta = \beta - \frac{1}{2}\pi$. The reduced electric field, defined as $e = (E/E_0)^2$, where $E_0 = (\pi/d\sin\omega)(B_3/\varepsilon_0\varepsilon_a)^{1/2}$, should be used instead of h.

3.2.1. *Positive dielectric anisotropy*

The solutions are symmetric in two cases. If $\phi_0 = 0$ then $\alpha = \omega$ and the threshold field is

$$e_c = \frac{\sin \omega \, (b_1 \cos^2 \beta + \sin^2 \beta - b_{13} \sin 2\beta)}{\sin \theta \cos \beta}. \tag{35}$$

The deformation is possible for a suitable initial orientation of the smectic layers determined by the angle θ:

$$0 < \theta < \tfrac{1}{2}\pi - \omega \quad \text{or} \quad -\pi < \theta < -\omega - \tfrac{1}{2}\pi. \tag{36}$$

The transition is continuous if

$$\max(\delta_c - \omega + \tfrac{1}{2}\pi, 0) \leqslant \theta < \tfrac{1}{2}\pi - \omega \quad \text{or} \quad \max(\delta_c - \omega - \tfrac{1}{2}\pi, -\pi) \leqslant \theta < -\tfrac{1}{2}\pi - \omega \tag{37}$$

and discontinuous if

$$\min(\delta_c - \omega + \tfrac{1}{2}\pi, 0) < \theta < \max(\delta_c - \omega + \tfrac{1}{2}\pi, 0), \tag{38}$$

where δ_c is given by equation (27). If $\phi_0 = \frac{1}{2}\pi$, $\theta = 0$ and $\alpha = 0$, the transition is continuous and starts at $e_c = b_2$.

The other configurations, for which $\mathbf{E} \perp \mathbf{n}_0$, i.e. $\cos \phi_0 = \cot \omega \tan \alpha$, $\delta = \alpha - \frac{1}{2}\pi$ and $\theta = 0$, correspond to a planar orientation of director and tilted smectic layers. The deformation is of type 3 and the threshold is

$$e_c = b_2 + \cot^2 \phi_0 (\tan^2 \omega - 2b_{13} \tan \omega + b_1). \tag{39}$$

Type 4 deformation can be achieved if

$$\cos^2 \phi_0 = \frac{b_{13} \tan \omega - b_1}{\tan^2 \omega - b_{13} \tan \omega}, \tag{40}$$

$$\tan^2 \alpha \, [\sin^2 (\phi_0 - \gamma) - 3] + 2 \tan \alpha b_{13} \cos \phi_0 \, [3 - 5 \sin^2 (\phi_0 - \gamma)] + [3(b_1 + b_2) + b_1 \sin^2 \phi_0 + b_2 \cos^2 \phi_0] \sin^2 (\phi_0 - \gamma) - 3(b_1 \cos^2 \phi_0 + b_2 \sin^2 \phi_0) < 0. \tag{41}$$

The transition is continuous if equation (40) is valid together with the opposite sign of inequality (41). However, the condition (40) can be satisfied if $\tan \omega > b_1/b_{13}$, which seems to be impossible for real materials.

3.2.2. *Negative dielectric anisotropy*

The only critical point can occur if $\phi_0 = 0$. The threshold is given by

$$e_c = \frac{\sin \omega \, (b_1 \cos^2 \beta + \sin^2 \beta - b_{13} \sin 2\beta)}{\sin (\omega - \beta) \cos \beta}, \tag{42}$$

and is positive for $-\omega - \frac{1}{2}\pi < \theta < 0$ or $\frac{1}{2}\pi - \omega < \theta < \pi$. The deformation is always of type 1, since a_4 remains positive.

4. Concluding remarks

The results obtained in the approach presented have a qualitative character. The values of ξ_m are approximate but acceptable in the vicinity of critical points.

The approximation is due to limiting the $\phi(z)$ function to the first term of a Fourier series, (equation (3)), and to the truncation of the Taylor series of G at the lowest possible order. The function $\phi(z)$ has a topological form suitable for qualitative description of the real director distribution, but insufficient for numerically exact solutions, especially at high field strength. However, if the next term of Fourier expansion, for instance of the form $\psi_m \cos(3\pi z/d)$, is added to $\phi(z)$ then the free-energy density becomes a function of two variables ξ_m and ψ_m. Its Taylor expansion contains many more terms than in the single-variable case. Determination of the catastrophe, which is equivalent to such a function, is also much more laborious, whereas the results remain approximate. Since all the interesting qualitative features of the final solutions can be obtained by using the simple form (3), it seems reasonable to exclude higher terms during calculations.

The configurations considered in §3 give rise to a critical behaviour due to $a_1 = 0$. If this condition is not satisfied then there is no threshold. The deformation starts at $h = 0$ and increases smoothly. In some cases hysteresis is also possible; examples of such behaviour are shown in figure 3.

The values of particular elastic constants of the smectic C phase have not been determined experimentally. We may suppose that relations between them are similar to the relations between corresponding elastic constants of nematics describing the analogous field effects. According to this assumption, the values of b_1 and b_2 were postulated, a small value of b_{13} was chosen.

In some configurations considered in this paper two minima of the free energy exist at the same field strength. In such situations the nematic liquid-crystal layers behave according to the delay convention, i.e. they remain in one equilibrium state while this state exists. In systems in which fluctuations prevent the stay in metastable state another possibility is realized; the transition occurs when the free energies of both states are equal. This principle is called the Maxwell convention. In the smectic C phase, the agreement with the delay principle was found experimentally by Meirovitch *et al.* [3]. Occurrence of both types of behaviour has been suggested in experiments performed by Pelzl *et al.* [5]. The results presented here were obtained according to the delay convention. The detailed calculation of the threshold field due to the Maxwell convention is not possible, as the energies of the deformed states are known only approximately. The approach presented should be valid for layers in which the deviations of director orientation, able to give rise to switching to another equilibrium state, are absent.

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